

FYJC - MATHEMATICS & STATISTICS

PAPER - I

ANGLE MEASUREMENTS

EXERCISE - 1

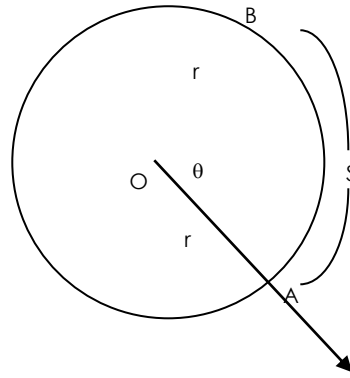
01. sum of measures of two angles is 130° and difference is $\frac{5\pi}{18}^c$. Find the angle in radians .
ans : $\frac{\pi}{2}^c ; 2\frac{\pi}{9}^c$
02. sum of measures of two angles is 100° and difference is $\frac{2\pi}{9}^c$. Find the angle in radians .
ans : $\frac{7\pi}{18}^c ; \frac{\pi}{6}^c$
03. if the radian measure of the two angles of a triangle are as given below , find the radian measure and the degree measure of the third angle
a) $\frac{5\pi}{9}^c ; \frac{5\pi}{18}^c$ a) $\frac{3\pi}{5}^c ; \frac{4\pi}{15}^c$
ans : a) $30^\circ ; \frac{\pi}{6}^c$ a) $24^\circ ; 2\frac{\pi}{15}^c$
04. difference between two acute angles of a right angled triangle is $\frac{2\pi}{5}^c$. Find them in radians
ans : $\frac{9\pi}{20}^c ; \frac{\pi}{20}^c$
05. difference between two acute angles of a right angled triangle is $\frac{3\pi}{10}^c$. Find them in radians
ans : $\frac{2\pi}{5}^c ; \frac{\pi}{10}^c$
06. In $\triangle ABC$, $B = 100^\circ$, $C = \frac{7\pi}{36}^c$. Find A in degrees and radians
ans : $45^\circ ; \frac{\pi}{4}^c$
07. angles of triangle are in ratio 3 : 7 : 8 . Find their measure in degree and radians
ans : $30^\circ , 70^\circ , 80^\circ$; $\frac{\pi}{6}^c , \frac{7\pi}{18}^c , \frac{4\pi}{9}^c$
08. angles of quadrilateral are in ratio 2 : 3 : 5 : 8 . Find their measure in radians
ans : $\frac{2\pi}{9}^c , \frac{\pi}{3}^c , \frac{5\pi}{9}^c , \frac{8\pi}{9}^c$
09. angles of quadrilateral are in ratio 3 : 4 : 5 : 6 . Find measure in radians
ans : $\frac{\pi}{3}^c , \frac{4\pi}{9}^c , \frac{5\pi}{9}^c , \frac{2\pi}{3}^c$
10. one angle of triangle has measure $\frac{2\pi}{9}^c$ and measure of other two angles are in ratio 4 : 3 . Find their measure in degree and radians
ans : $80^\circ , 60^\circ$; $\frac{4\pi}{9}^c , \frac{\pi}{3}^c$
11. one angle of quadrilateral has measure $\frac{2\pi}{5}^c$ and measure of other three angles are in ratio 2 : 3 : 4 . Find their measure in degree and radians
ans : $64^\circ , 96^\circ , 128^\circ$; $\frac{16\pi}{45}^c , \frac{24\pi}{45}^c , \frac{32\pi}{45}^c$

- 12.** one angle of quadrilateral has measure $2\pi/9^c$ and measure of other three angles are in ratio 3 : 5 : 8 .
Find their measure in degree and radians
ans : $60^\circ, 100^\circ, 160^\circ$; $\pi/3^c, 5\pi/9^c, 8\pi/9^c$
- 13.** if one angle of the quadrilateral has measure $3\pi/10$ radian and the measure of other 3 angles are in the ratio 4 : 7 : 6 , find their measure in degrees (SEP – 2012 ; $72^\circ, 126^\circ, 108^\circ$)
- 14.** If one of the exterior angle of triangle is $2\pi/3$ and the ratio of remaining two interior angle is 2:3 . Find the angles in degree and radians ;
ans : $4\pi/15^c, 2\pi/5^c, \pi/3^c$
- 15.** the exterior angle of one angle of a triangle is 100° . The difference in the measure of the two remaining angles of the triangle is half the measure of the given exterior angle . Find the measures of the remaining angles of the triangle in radians
ans : $5\pi/12^c ; 5\pi/36^c$
- 16.** the angles of a triangle are in AP and the greatest angle is 84° . Find all the three angles
ans: $7\pi/15^c, \pi/5^c, \pi/3^c$
- 17.** the angles of a triangle are in A.P. and the ratio of the degree measure of the least to the radian measure of the greatest is $60: \pi$. Find the angles of the triangle in degrees & radians
- 18.** the angles of a quadrilateral are in AP and the greatest angle is double the least . Express the least angle in radians
ans: $\pi/3^c$
- 19.** if $x^c = 405^\circ$ and $y^\circ = \frac{-\pi^c}{12}$, find x and y
- 20.** if $\theta^\circ = -\frac{5\pi^c}{9}$ and $\phi^c = 900^\circ$ find θ & ϕ
- 21.** Find the measure of interior angle of the regular polygon
- | | | | |
|---------------------|--------------------|--------------------|-----------------------------|
| 01. Pentagon | 02. Hexagon | 03. Octagon | 04. 10 sided polygon |
| ans : $3\pi/5^c$ | ans : $2\pi/3^c$ | ans : $3\pi/4^c$ | ans : $4\pi/5^c$ |
- 22.** Find the number of sides of a regular polygon if each of its interior angles is $3\pi/4^c$.
ans : 8 .
- 23.** Find the number of sides of a regular polygon if each of its interior angles is $4\pi/5^c$.
ans : 10 .

EXERCISE - 2 ARC LENGTH & AREA OF SECTOR

a) if S is the length of an arc of a circle of radius r which subtends an angle θ at the centre of the circle measured in radians then $S = r\theta$

b) if θ , in radian is an angle between two radii of circle with radius r then the area of the corresponding sector is $\frac{1}{2}r^2\theta$



01. find the length of arc if a circle of diameter 10 cm if arc is subtending an angle of 36° at the centre
ans : π cm
02. find the length of arc of circle which subtends 108° at centre in a circle of radius 15 cm
ans : 9π cm
03. radius of circle is 9 cm . Find the length of arc of this circle which cuts off a chord of length equal to length of radius **ans :** 3π cm
04. in a circle of diameter 40 cm , the length of chord is 20 cm . Find the length of minor arc of the chord
ans : $20\pi/3$ cm
05. a pendulum 14 cm long oscillates through an angle of 18° . Find the length of path described by its extremity
ans : $7\pi/5$ cm
06. a wire of length 10 cms is bent so as to form an arc of a circle of radius 4 cms . What is the angle subtended at the center in degrees
ans : $(450/\pi)^\circ$
07. area of circle is 25π sq. cm . Find the length of arc subtending an angle of 144° at center . Also find the area of the corresponding sector
ans : 4π cm ; 10π sq.cm
08. area of circle is 81π sq. cm . Find the length of arc subtending an angle of 150° at center . Also find the area of the corresponding sector
ans : $15\pi/2$ cm ; $135\pi/4$ sq.cm

09. area of circle is 81π sq. cm . Find the length of arc subtending and angle of 300° at center . Also find the area of the corresponding sector
ans : 15π cm ; 135π sq.cm
10. Perimeter of sector of a circle of area 25π sq. cm is 20 cm . Find the area of the sector
ans : 25 sq.cm
11. Perimeter of sector of a circle of area 64π sq. cm is 56 cm . Find the area of the sector
ans : 160 sq.cm
12. Perimeter of sector of a circle is 4 times radius of circle . Find the central angle
ans : 2°
13. Horse is tied to a post by a rope . If the horse moves circular path , always keeping rope tight and describes an arc of length 88 m when it traces angle of 72° at centre . Find length of rope ($\pi = 22/7$)
ans : 70 m
14. ΔPQR is equilateral Δ with side 18 cm . A circle is drawn on segment QR as diameter . Find length of arc of this circle intercepted within triangle
ans : 3π cm
15. if the arcs of the same lengths in two circles subtend angles 65° and 110° at the centres , find the ratio of their radii
ans : 22 : 13
16. In a circle of radius 12 cms , an arc PQ subtends the angle of 30° at the centre . Find the area between the arc PQ and chord PQ
ans : $12(\pi - 3)$ sq. cms
17. OAB is a sector of a circle with center O and radius 12 cms . If $m\angle AOB = 60^\circ$ find the difference between the area of sector AOB and area of ΔAOB
ans : $12(2\pi - 3\sqrt{3})$ sq. cms

SOLUTION TO EXERCISE - 1

01. sum of measures of two angles is 130° and difference is $5\pi/18^c$. Find the angle in radians .

$$\text{SOLUTION : } 5\pi/18^c = \frac{5(180)}{18} = 50^\circ$$

$$\begin{array}{rcl} \angle A + \angle B & = & 130^\circ \\ \angle A - \angle B & = & 50^\circ \\ \hline 2\angle A & = & 180^\circ \\ \angle A & = & 90^\circ \end{array}$$

$$\begin{aligned} \angle A &= 90 \times \frac{\pi}{180} \\ &= \frac{\pi^c}{2} \end{aligned}$$

$$\begin{array}{rcl} \angle A + \angle B & = & 130^\circ \\ \angle A - \angle B & = & 50^\circ \\ \hline 2\angle B & = & 80^\circ \\ \angle B & = & 40^\circ \end{array}$$

$$\begin{aligned} \angle B &= 40 \times \frac{\pi}{180} \\ &= \frac{2\pi^c}{9} \end{aligned}$$

02. sum of measures of two angles is 100° and difference is $2\pi/9^c$. Find the angle in radians .

$$\text{SOLUTION : } 2\pi/9^c = \frac{2(180)}{9} = 40^\circ$$

$$\begin{array}{rcl} \angle A + \angle B & = & 100^\circ \\ \angle A - \angle B & = & 40^\circ \\ \hline 2\angle A & = & 140^\circ \\ \angle A & = & 70^\circ \end{array}$$

$$\begin{aligned} \angle A &= 70 \times \frac{\pi}{180} \\ &= \frac{7\pi^c}{18} \end{aligned}$$

$$\begin{array}{rcl} \angle A + \angle B & = & 100^\circ \\ \angle A - \angle B & = & 40^\circ \\ \hline 2\angle B & = & 60^\circ \\ \angle B & = & 30^\circ \end{array}$$

$$\begin{aligned} \angle B &= 30 \times \frac{\pi}{180} \\ &= \frac{\pi^c}{6} \end{aligned}$$

03. if the radian measure of the two angles of a triangle are as given below , find the radian measure and the degree measure of the third angle

$$\text{a) } 5\pi/9^c ; 5\pi/18^c \qquad \text{a) } 3\pi/5^c ; 4\pi/15^c$$

$$\text{SOLN a) : } 5\pi/18^c = \frac{5(180)}{18} = 50^\circ$$

$$5\pi/9^c = \frac{5(180)}{9} = 100^\circ$$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$100^\circ + 50^\circ + \angle C = 180^\circ$$

$$\angle C = 30^\circ$$

$$= 30 \times \frac{\pi}{180}$$

$$= \frac{\pi^c}{6}$$

$$\text{SOLN b) : } 3\pi/5^c = \frac{3(180)}{5} = 108^\circ$$

$$4\pi/15^c = \frac{4(180)}{15} = 48^\circ$$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$108^\circ + 48^\circ + \angle C = 180^\circ$$

$$\angle C = 24^\circ$$

$$= 24 \times \frac{\pi}{180}$$

$$= \frac{2\pi^c}{15}$$

04. difference between two acute angles of a right angled triangle is $2\pi/5^c$. Find them in radians

$$\text{SOLUTION : } 2\pi/5^c = \frac{2(\cancel{180})}{5} = 72^\circ$$

$\begin{aligned} \angle A + \angle B &= 90^\circ \\ \angle A - \angle B &= 72^\circ \\ \hline 2\angle A &= 162^\circ \\ \angle A &= 81^\circ \end{aligned}$ $\angle A = \cancel{81} \times \frac{\pi}{\cancel{180} 20} = \frac{9\pi^c}{20}$	$\begin{aligned} \angle A + \angle B &= 90^\circ \\ \angle A - \angle B &= 72^\circ \\ \hline 2\angle B &= 18^\circ \\ \angle B &= 9^\circ \end{aligned}$ $\angle B = \cancel{9} \times \frac{\pi}{\cancel{180} 20} = \frac{\pi^c}{20}$
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05. difference between two acute angles of a right angled triangle is $3\pi/10^c$. Find them in radians

$$\text{SOLUTION : } 3\pi/10^c = \frac{3(\cancel{180})}{10} = 54^\circ$$

$\begin{aligned} \angle A + \angle B &= 90^\circ \\ \angle A - \angle B &= 54^\circ \\ \hline 2\angle A &= 114^\circ \\ \angle A &= 57^\circ \end{aligned}$ $\angle A = \cancel{57} \times \frac{\pi}{\cancel{180} 20} = \frac{19\pi^c}{20}$	$\begin{aligned} \angle A + \angle B &= 90^\circ \\ \angle A - \angle B &= 54^\circ \\ \hline 2\angle B &= 36^\circ \\ \angle B &= 18^\circ \end{aligned}$ $\angle B = \cancel{18} \times \frac{\pi}{\cancel{180} 20} = \frac{3\pi^c}{10}$
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06. In $\triangle ABC$, $B = 100^\circ$, $C = 7\pi/36^c$. Find A in degrees and radians

$$\text{SOLUTION : } 2\pi/5^c = \frac{7(\cancel{180})}{36} = 35^\circ$$

$$\begin{aligned} \angle A + \angle B + \angle C &= 180^\circ \\ \angle A + 100^\circ + 35^\circ &= 180^\circ \\ \angle A + 135^\circ &= 180^\circ \\ \angle A &= 45^\circ \\ &= 45 \times \frac{\pi}{180} \\ &= \frac{\pi^c}{4} \end{aligned}$$

07. angles of triangle are in ratio 3 : 7 : 8 . Find their measure in degree and radians

SOLUTION :

$$\begin{aligned} \text{Let } \angle A &= 3x ; \angle B = 7x ; \angle C = 8x \\ \angle A + \angle B + \angle C &= 180^\circ \\ 3x + 7x + 8x &= 180^\circ \\ 18x &= 180^\circ \\ x &= 10^\circ \end{aligned}$$

$\begin{aligned} \angle A &= 3x \\ &= 3(10) \\ &= 30 \\ &= 30 \times \frac{\pi}{180} \\ &= \frac{\pi^c}{6} \end{aligned}$	$\begin{aligned} \angle B &= 7x \\ &= 7(10) \\ &= 70 \\ &= 70 \times \frac{\pi}{180} \\ &= \frac{7\pi^c}{18} \end{aligned}$	$\begin{aligned} \angle C &= 8x \\ &= 8(10) \\ &= 80 \\ &= 80 \times \frac{\pi}{180} \\ &= \frac{4\pi^c}{9} \end{aligned}$
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08. angles of quadrilateral are in ratio 2 : 3 : 5 : 8 . Find their measure in radians

SOLUTION :

$$\text{Let } \angle A = 2x ; \angle B = 3x ; \angle C = 5x ; \angle D = 8x$$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$2x + 3x + 5x + 8x = 360^\circ$$

$$18x = 360^\circ$$

$$x = 20^\circ$$

$$\angle A = 2x = 2(20^\circ) = 40^\circ = 40 \times \frac{\pi}{180} = \frac{2\pi}{9}^c$$

$$\angle B = 3x = 3(20^\circ) = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3}^c$$

$$\angle C = 5x = 5(20^\circ) = 100^\circ = 100 \times \frac{\pi}{180} = \frac{5\pi}{9}^c$$

$$\angle D = 8x = 8(20^\circ) = 160^\circ = 160 \times \frac{\pi}{180} = \frac{8\pi}{9}^c$$

09. angles of quadrilateral are in ratio 3:4:5:6 . Find measure in radians

SOLUTION :

$$\text{Let } \angle A = 3x ; \angle B = 4x ; \angle C = 5x ; \angle D = 6x$$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$3x + 4x + 5x + 6x = 360^\circ$$

$$18x = 360^\circ$$

$$x = 20^\circ$$

$$\angle A = 3x = 3(20^\circ) = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3}^c$$

$$\angle B = 4x = 4(20^\circ) = 80^\circ = 80 \times \frac{\pi}{180} = \frac{4\pi}{9}^c$$

$$\angle C = 5x = 5(20^\circ) = 100^\circ = 100 \times \frac{\pi}{180} = \frac{5\pi}{9}^c$$

$$\angle D = 8x = 6(20^\circ) = 120^\circ = 120 \times \frac{\pi}{180} = \frac{2\pi}{3}^c$$

10. one angle of triangle has measure $\frac{2\pi}{9}^c$ and measure of other two angles are in ratio 4 : 3 . Find their measure in degree and radians

SOLUTION :

$$\frac{2\pi}{9} = \frac{2(180)}{9} = 40^\circ$$

$$\text{Let } \angle A = 40^\circ ; \angle B = 4x ; \angle C = 3x$$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$40 + 4x + 3x = 180^\circ$$

$$7x = 140^\circ$$

$$x = 20^\circ$$

$$\angle B = 4x = 4(20^\circ) = 80^\circ = 80 \times \frac{\pi}{180} = \frac{4\pi}{9}^c$$

$$\angle C = 3x = 3(20^\circ) = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3}^c$$

11. one angle of quadrilateral has measure $2\pi/5^c$ and measure of other three angles are in ratio 2 : 3 : 4 . Find their measure in degree and radians

SOLUTION : $\frac{36}{5}$

$$\frac{2\pi}{5} = \frac{2(+80)}{5} = 72^\circ$$

$$\text{Let } \angle A = 72^\circ ; \angle B = 2x ; \angle C = 3x ; \angle D = 4x$$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$72^\circ + 2x + 3x + 4x = 360^\circ$$

$$9x = 288^\circ$$

$$x = 32^\circ$$

$$\angle B = 2x = 2(32^\circ) = 64^\circ = 64 \times \frac{\pi}{180} = \frac{16\pi}{45}^c$$

$$\angle C = 3x = 3(32^\circ) = 96^\circ = 96 \times \frac{\pi}{180} = \frac{8\pi}{15}^c$$

$$\angle D = 4x = 4(32^\circ) = 128^\circ = 128 \times \frac{\pi}{180} = \frac{32\pi}{45}^c$$

12. one angle of quadrilateral has measure $2\pi/9^c$ and measure of other three angles are in ratio 3 : 5 : 8 . Find their measure in degree and radians

SOLUTION :

$$\frac{2\pi}{9} = \frac{2(+80)}{9} = 40^\circ$$

$$\text{Let } \angle A = 40^\circ ; \angle B = 3x ; \angle C = 5x ; \angle D = 8x$$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$40^\circ + 3x + 5x + 8x = 360^\circ$$

$$16x = 320^\circ \quad x = 20^\circ$$

$$\angle B = 3x = 3(20^\circ) = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3}^c$$

$$\angle C = 5x = 5(20^\circ) = 100^\circ = 100 \times \frac{\pi}{180} = \frac{5\pi}{9}^c$$

$$\angle D = 8x = 8(20^\circ) = 160^\circ = 160 \times \frac{\pi}{180} = \frac{8\pi}{9}^c$$

13. if one angle of the quadrilateral has measure $3\pi/10$ radian and the measure of other 3 angles are in the ratio 4 : 7 : 6 , find their measure in degrees (**SEP – 2012** ; 72° , 126° , 108°)

SOLUTION :

$$\frac{3\pi}{10} = \frac{3(180)}{10} = 54^\circ$$

$$\text{Let } \angle A = 54^\circ ; \angle B = 4x ; \angle C = 7x ; \angle D = 6x$$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$54^\circ + 4x + 7x + 6x = 360^\circ$$

$$17x = 306^\circ$$

$$x = 18^\circ$$

$$\angle B = 4x = 4(18^\circ) = 72^\circ = \frac{72}{180} \times \frac{\pi}{1} = \frac{2\pi}{5}^c$$

$$\angle C = 7x = 7(18^\circ) = 126^\circ = \frac{126}{180} \times \frac{\pi}{1} = \frac{7\pi}{10}^c$$

$$\angle D = 6x = 6(18^\circ) = 108^\circ = \frac{108}{180} \times \frac{\pi}{1} = \frac{3\pi}{5}^c$$

14. If one of the exterior angle of triangle is $2\pi/3$ and the ratio of remaining two interior angle is 2:3 . Find the angles in degree and radians ;

SOLUTION :

$$\frac{2\pi}{3} = \frac{2(180)}{3} = 120^\circ$$

Let the remaining two interior angles be $\angle A$ & $\angle B$

Let $\angle A = 2x$; $\angle B = 3x$

$$\angle A + \angle B = 120^\circ \quad \dots\dots\dots \text{exterior angle theorem}$$

$$2x + 3x = 120^\circ$$

$$5x = 120^\circ$$

$$x = 24^\circ$$

$$\angle A = 2x = 2(24^\circ) = 48^\circ = \frac{48}{180} \times \frac{\pi}{1} = \frac{4\pi}{15}^c$$

$$\angle B = 3x = 3(24^\circ) = 72^\circ = \frac{72}{180} \times \frac{\pi}{1} = \frac{2\pi}{5}^c$$

15. the exterior angle of one angle of a triangle is 100° . The difference in the measure of the two remaining angles of the triangle is half the measure of the given exterior angle . Find the measures of the remaining angles of the triangle in radians

SOLUTION :

Let the remaining two interior angles be $\angle A$ & $\angle B$

$$\angle A + \angle B = 100^\circ \quad \dots\dots\dots \text{exterior angle theorem}$$

$$\begin{array}{r} \angle A + \angle B = 100^\circ \\ \angle A - \angle B = 50^\circ \\ \hline 2\angle A = 150^\circ \\ \angle A = 75^\circ \\ \angle A = \frac{75}{180} \times \frac{\pi}{1} \\ = \frac{5\pi}{12}^c \end{array}$$

$$\begin{array}{r} \angle A + \angle B = 100^\circ \\ \angle A - \angle B = 50^\circ \\ \hline 2\angle B = 50^\circ \\ \angle B = 25^\circ \\ \angle B = \frac{25}{180} \times \frac{\pi}{1} \\ = \frac{5\pi}{36}^c \end{array}$$

16. the angles of a triangle are in AP and the greatest angle is 84° . Find all the three angles

SOLUTION :

Let $\angle A = a - d$; $\angle B = a$; $\angle C = a + d$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$a - d + a + a + d = 180^\circ$$

$$3a = 180^\circ$$

$$a = 60^\circ$$

also given $\angle C = 84^\circ$

$$a + d = 84^\circ$$

$$60 + d = 84 \quad \quad \quad d = 24^\circ$$

$$\angle A = a - d = 60 - 24 = 36^\circ = 36 \times \frac{\pi}{180} = \frac{\pi}{5}^c$$

$$\angle B = a = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3}^c$$

$$\angle C = a + d = 60 + 24 = 84^\circ = 84 \times \frac{\pi}{180} = \frac{7\pi}{15}^c$$

17. the angles of a triangle are in A.P. and the ratio of the degree measure of the least to the radian measure of the greatest is $60: \pi$.

Find the angles of the triangle in degrees & radians

SOLUTION :

$$\text{Let } \angle A = a - d ; \quad \angle B = a ; \quad \angle C = a + d$$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$a - d + a + a + d = 180^\circ$$

$$3a = 180^\circ$$

$$a = 60^\circ$$

$$\text{also given } \angle A : \angle C = 60: \pi$$

$$a - d : a + d = 60: 180$$

$$\frac{a - d}{a + d} = \frac{1}{3}$$

$$3a - 3d = a + d$$

$$2a = 4d$$

$$a = 2d$$

$$60 = 2d \quad d = 30^\circ$$

$$\angle A = a - d = 60 - 30 = 30^\circ = 30 \times \frac{\pi}{180} = \frac{\pi}{6}^c$$

$$\angle B = a = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3}^c$$

$$\angle C = a + d = 60 + 30 = 90^\circ = 90 \times \frac{\pi}{180} = \frac{\pi}{2}^c$$

18. the angles of a quadrilateral are in AP and the greatest angle is double the least. Express the least angle in radians

SOLUTION :

$$\text{Let } \angle A = a - 3d ; \angle B = a - d ; \quad \angle C = a + d ; \angle D = a + 3d$$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$a - 3d + a - d + a + d + a + 3d = 360^\circ$$

$$4a = 360^\circ$$

$$a = 90^\circ$$

$$\text{also given : } a + 3d = 2(a - 3d)$$

$$a + 3d = 2a - 6d$$

$$9d = a$$

$$9d = 90$$

$$d = 10$$

$$\angle A = a - 3d = 90 - 30 = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3}^c$$

$$\angle B = a - d = 90 - 10 = 80^\circ = 80 \times \frac{\pi}{180} = \frac{4\pi}{9}^c$$

$$\angle C = a + d = 90 + 10 = 100^\circ = 100 \times \frac{\pi}{180} = \frac{5\pi}{9}^c$$

$$\angle D = a + 3d = 90 + 30 = 120^\circ = 120 \times \frac{\pi}{180} = \frac{2\pi}{3}^c$$

19. if $x^{\circ} = 405^{\circ}$ and $y^{\circ} = \frac{-\pi^{\circ}}{12}$, find x and y

SOLUTION :

$$x^{\circ} = 405^{\circ} = 405 \times \frac{\pi}{180} = \frac{9\pi}{4}^{\circ}$$

$$y^{\circ} = \frac{-\pi^{\circ}}{12} = \frac{-180}{12} = -15^{\circ} \quad x = 9\pi/4 \text{ \& } y = -15$$

20. if $\theta^{\circ} = -\frac{5\pi^{\circ}}{9}$ and $\phi^{\circ} = 900^{\circ}$ find θ & ϕ

SOLUTION :

$$\theta^{\circ} = -\frac{5\pi^{\circ}}{9} = \frac{-5(180)}{9} = -100^{\circ}$$

$$\phi^{\circ} = 900^{\circ} = 900 \times \frac{\pi}{180} = 5\pi^{\circ}$$

NOTE

❖ SUM OF MEASURES OF EXTERIOR ANGLES IN A REGULAR POLYGON = 360°

❖ MEASURE OF ONE EXTERIOR ANGLE = $\frac{360^{\circ}}{\text{SIDES OF POLYGON}}$

❖ MEASURE OF INTERIOR ANGLE = $180^{\circ} - \text{EXT. ANGLE}$

21. Find the measure of interior angle of the regular polygon

01. Pentagon

SOLUTION :

$$n = 5$$

$$\text{sum of exterior } \angle\text{'s} = 360^{\circ}$$

$$m(\text{ exterior } \angle) = \frac{360}{n} = \frac{360}{5} = 72^{\circ}$$

$$m(\text{ interior } \angle) = 180 - 72 = 108^{\circ}$$

$$= 108 \times \frac{\pi}{180} = \frac{3\pi}{5}^{\circ}$$

02. Hexagon

SOLUTION :

$$n = 6$$

$$\text{sum of exterior } \angle\text{'s} = 360^{\circ}$$

$$m(\text{ exterior } \angle) = \frac{360}{n} = \frac{360}{6} = 60^{\circ}$$

$$m(\text{ interior } \angle) = 180 - 60 = 120^{\circ}$$

$$= 120 \times \frac{\pi}{180} = \frac{2\pi}{3}^{\circ}$$

03. Octagon**SOLUTION :**

$$n = 8$$

$$\text{sum of exterior } \angle\text{'s} = 360^\circ$$

$$m(\text{ exterior } \angle) = \frac{360}{n} = \frac{360}{8} = 45^\circ$$

$$\begin{aligned} m(\text{ interior } \angle) &= 180 - 45 = 135^\circ \\ &= 135 \times \frac{\pi}{180} = \frac{3\pi}{4} \text{ c} \end{aligned}$$

04. 10 sided polygon**SOLUTION :**

$$n = 10$$

$$\text{sum of exterior } \angle\text{'s} = 360^\circ$$

$$m(\text{ exterior } \angle) = \frac{360}{n} = \frac{360}{10} = 36^\circ$$

$$\begin{aligned} m(\text{ interior } \angle) &= 180 - 36 = 144^\circ \\ &= 144 \times \frac{\pi}{180} = \frac{4\pi}{5} \text{ c} \end{aligned}$$

22. Find the number of sides of a regular polygon if each of its interior angles is $3\pi/4$ c .

SOLUTION :

$$\frac{3\pi}{4} = \frac{3(180)}{4} = 135^\circ$$

$$m(\text{ interior } \angle) = 135^\circ$$

$$m(\text{ exterior } \angle) = 180 - 135 = 45^\circ$$

$$\text{sum of exterior } \angle\text{'s} = 360^\circ$$

$$m(\text{ exterior } \angle) = \frac{360}{n}$$

$$45 = \frac{360}{n}$$

$$n = \frac{360}{45} = 8$$

23. Find the number of sides of a regular polygon if each of its interior angles is $4\pi/5$ c .

SOLUTION :

$$\frac{4\pi}{5} = \frac{4(180)}{5} = 144^\circ$$

$$m(\text{ interior } \angle) = 144^\circ$$

$$m(\text{ exterior } \angle) = 180 - 144 = 36^\circ$$

$$\text{sum of exterior } \angle\text{'s} = 360^\circ$$

$$m(\text{ exterior } \angle) = \frac{360}{n}$$

$$36 = \frac{360}{n}$$

$$n = \frac{360}{36} = 10$$

SOLUTION TO EXERCISE - 2

01.

find the length of arc in a circle of diameter 10 cm if arc is subtending an angle of 36° at the centre

SOLUTION :

$$r = 10/2 = 5\text{cm}$$

$$\theta = 36^\circ$$

$$= 36 \times \frac{\pi}{180}$$

$$= \frac{\pi}{5}^c$$

$$s = r\theta$$

$$= 5 \times \frac{\pi}{5}$$

$$= \pi \text{ cm}$$

02.

find the length of arc of circle which subtends 108° at centre in a circle of radius 15 cm

SOLUTION :

$$r = 15 \text{ cm}$$

$$\theta = 108^\circ$$

$$= 108 \times \frac{\pi}{180}$$

$$= \frac{3\pi}{5}^c$$

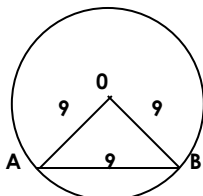
$$s = r\theta$$

$$= 15 \times \frac{3\pi}{5}$$

$$= 9\pi \text{ cm}$$

03.

radius of circle is 9 cm . Find the length of arc of this circle which cuts off a chord of length equal to length of radius



ΔOAB is an equilateral Δ

SOLUTION

$$r = 9 \text{ cm}$$

$$\theta = 60^\circ$$

$$= 60 \times \frac{\pi}{180}$$

$$= \frac{\pi}{3}^c$$

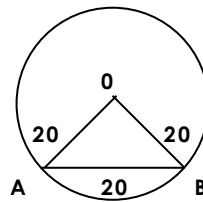
$$s = r\theta$$

$$= 9 \times \frac{\pi}{3}$$

$$= 3\pi \text{ cm}$$

04.

in a circle of diameter 40 cm , the length of chord is 20 cm . Find the length of minor arc of the chord



ΔOAB is an equilateral Δ

SOLUTION

$$r = 40/2 = 20 \text{ cm}$$

$$\theta = 60^\circ$$

$$= 60 \times \frac{\pi}{180}$$

$$= \frac{\pi}{3}^c$$

$$s = r\theta$$

$$= 20 \times \frac{\pi}{3}$$

$$= \frac{20\pi}{3} \text{ cm}$$

05.

a pendulum 14 cm long oscillates through an angle of 18° . Find the length of path described by its extremity

SOLUTION

$$r = 14 \text{ cm}$$

$$\theta = 18^\circ$$

$$= 18 \times \frac{\pi}{180}$$

$$= \frac{\pi}{10}^{\circ}$$

$$s = r\theta$$

$$= 14 \times \frac{\pi}{10}$$

$$= \frac{7\pi}{5} \text{ cm}$$

06.

a wire of length 10 cms is bent so as to form an arc of a circle of radius 4 cms . What is the angle subtended at the center in degrees

SOLUTION :

$$s = 10 \text{ cm}$$

$$r = 4 \text{ cm}$$

$$s = r\theta$$

$$10 = 4\theta$$

$$\theta = \frac{10}{4}$$

$$= \frac{5}{2} \times \frac{180}{\pi}$$

$$= \frac{450}{\pi}^{\circ}$$

07.

area of circle is 25π sq. cm . Find the length of arc subtending and angle of 144° at center . Also find the area of the corresponding sector

SOLUTION

$$\checkmark \text{ area of circle} = 25\pi$$

$$\pi r^2 = 25\pi$$

$$r^2 = 25$$

$$r = 5 \text{ cm}$$

$$\checkmark \theta = 144^{\circ}$$

$$= 144 \times \frac{\pi}{180}$$

$$= \frac{4\pi}{5}^{\circ}$$

$$\checkmark s = r\theta$$

$$= 5 \times \frac{4\pi}{5}$$

$$= 4\pi \text{ cm}$$

 \checkmark area of sector

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \cdot 25 \cdot \frac{4\pi}{5}$$

$$= 10\pi \text{ sq.cm}$$

08.

area of circle is 81π sq. cm . Find the length of arc subtending and angle of 150° at center . Also find the area of the corresponding sector

SOLUTION :

$$\checkmark \text{ area of circle} = 81\pi$$

$$\pi r^2 = 81\pi$$

$$r^2 = 81$$

$$r = 9 \text{ cm}$$

$$\checkmark \theta = 150^{\circ}$$

$$= 150 \times \frac{\pi}{180}$$

$$= \frac{5\pi}{6}^{\circ}$$

$$\checkmark s = r\theta$$

$$= 9 \times \frac{5\pi}{6}$$

$$= \frac{15\pi}{2} \text{ cm}$$

$$\checkmark \text{ area of sector}$$

$$= \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \cdot 81 \cdot \frac{5\pi}{6}$$

$$= \frac{135\pi}{4} \text{ sq.cm}$$

09.

area of circle is 81π sq. cm . Find the length of arc subtending and angle of 300° at center . Also find the area of the corresponding sector

SOLUTION

$$\checkmark \text{ area of circle} = 81\pi$$

$$\pi r^2 = 81\pi$$

$$r^2 = 81$$

$$r = 9 \text{ cm}$$

$$\checkmark \theta = 300^\circ$$

$$= 300 \times \frac{\pi}{180}$$

$$= \frac{5\pi}{3}^\circ$$

$$\checkmark s = r\theta$$

$$= 9 \times \frac{5\pi}{3}$$

$$= 15\pi \text{ cm}$$

$$\checkmark \text{ area of sector}$$

$$= \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \cdot 81 \cdot \frac{5\pi}{3}$$

$$= \frac{135\pi}{2} \text{ sq.cm}$$

10.

Perimeter of sector of a circle of area 25π sq. cm is 20 cm . Find the area of the sector

SOLUTION

$$\checkmark \text{ area of circle} = 25\pi$$

$$\pi r^2 = 25\pi$$

$$r^2 = 25$$

$$r = 5 \text{ cm}$$

$$\checkmark \text{ Perimeter of sector} = 20$$

$$2r + s = 20$$

$$10 + s = 20$$

$$s = 10 \text{ cm}$$

$$\checkmark s = r\theta$$

$$10 = 5\theta$$

$$\theta = 2^\circ$$

$$\checkmark \text{ area of sector}$$

$$= \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \cdot 25 \cdot 2$$

$$= 25 \text{ sq. cm}$$

11.

Perimeter of sector of a circle of area 64π sq. cm is 56 cm . Find the area of the sector

SOLUTION

$$\checkmark \text{ area of circle} = 64\pi$$

$$\pi r^2 = 64\pi$$

$$r^2 = 64$$

$$r = 8 \text{ cm}$$

$$\checkmark \text{ Perimeter of sector} = 56$$

$$2r + s = 56$$

$$16 + s = 56$$

$$s = 40 \text{ cm}$$

$$\checkmark s = r\theta$$

$$40 = 8\theta$$

$$\theta = 5^\circ$$

$$\checkmark \text{ area of sector}$$

$$= \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} \cdot 64 \cdot 5$$

$$= 160 \text{ sq. cm}$$

12.

Perimeter of sector of a circle is 4 times radius of circle . Find the central angle

SOLUTION

✓ Perimeter of sector = $4r$

$$2r + s = 4r$$

$$s = 2r$$

✓ $s = r\theta$

$$2r = r\theta$$

$$\theta = 2^\circ$$

13.

Horse is tied to a post by a rope . If the horse moves circular path , always keeping rope tight and describes an arc of length 88 m when it traces angle of 72° at centre . Find length of the rope ($\pi = 22/7$)

SOLUTION

$$s = 88 \text{ m}$$

$$\theta = 72^\circ$$

$$= 72 \times \frac{\pi}{180}$$

$$= \frac{2\pi}{5}$$

$$s = r\theta$$

$$88 = r \cdot \frac{2\pi}{5}$$

$$r = \frac{88 \times 5}{2\pi}$$

$$= \frac{88 \times 5}{2 \times \frac{22}{7}}$$

$$= 70 \text{ m}$$

15.

if the arcs of the same lengths in two circles subtend angles 65° and 110° at the centres , find the ratio of their radii

SOLUTION

✓ Circle – 1

$$r_1 = \text{Radius}$$

$$\theta_1 = 65^\circ$$

$$= 65 \times \frac{\pi}{180}$$

$$= \frac{65\pi}{180}$$

$$s_1 = \text{arc length.}$$

✓ Circle – 2

$$r_2 = \text{Radius}$$

$$\theta_2 = 110^\circ$$

$$= 110 \times \frac{\pi}{180}$$

$$= \frac{110\pi}{180}$$

$$s_2 = \text{arc length}$$

✓ $s_1 = s_2$ given

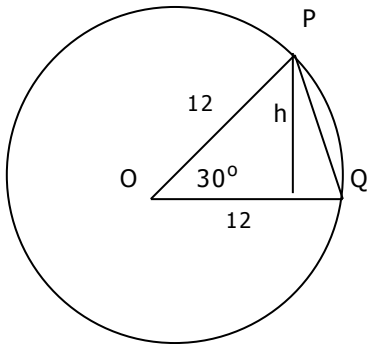
$$r_1 \cdot \theta_1 = r_2 \cdot \theta_2$$

$$r_1 \cdot \frac{65\pi}{180} = r_2 \cdot \frac{110\pi}{180}$$

$$\frac{r_1}{r_2} = \frac{110}{65} = \frac{22}{13}$$

16.

In a circle of radius 12 cms , an arc PQ subtends the angle of 30° at the centre . Find the area between the arc PQ and chord PQ



SOLUTION

$$\checkmark h = \frac{12}{2} = 6 \text{ cm}$$

$$\begin{aligned} \checkmark A(\Delta OPQ) &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 12 \times 6 \\ &= 36 \text{ sq. cm} \end{aligned}$$

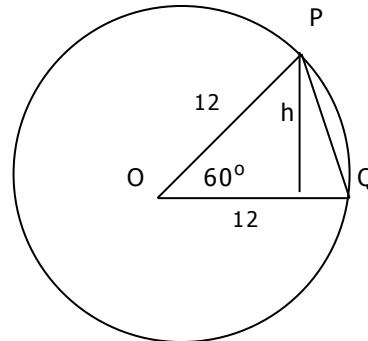
$$\begin{aligned} \checkmark \theta &= 30^\circ \\ &= 30 \times \frac{\pi}{180} \\ &= \frac{\pi}{6} \end{aligned}$$

$$\begin{aligned} \checkmark A(\text{sector OPQ}) &= \frac{1}{2} \cdot r^2 \cdot \theta \\ &= \frac{1}{2} \times 144 \times \frac{\pi}{6} \\ &= 12\pi \text{ sq. cm} \end{aligned}$$

$$\begin{aligned} \checkmark \text{ area between the arc PQ and chord PQ} &= A(\text{sector OPQ}) - A(\Delta OPQ) \\ &= 12\pi - 36 \\ &= 12(\pi - 3) \text{ sq. cm} \end{aligned}$$

17.

OAB is a sector of a circle with center O and radius 12 cms . If $m\angle AOB = 60^\circ$ find the difference between the area of sector AOB and area of ΔAOB



SOLUTION

$$\checkmark h = \frac{\sqrt{3} \times 12}{2} = 6\sqrt{3} \text{ cm}$$

$$\begin{aligned} \checkmark A(\Delta OPQ) &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 12 \times 6\sqrt{3} \\ &= 36\sqrt{3} \text{ sq. cm} \end{aligned}$$

$$\begin{aligned} \checkmark \theta &= 60^\circ \\ &= 60 \times \frac{\pi}{180} \\ &= \frac{\pi}{3} \end{aligned}$$

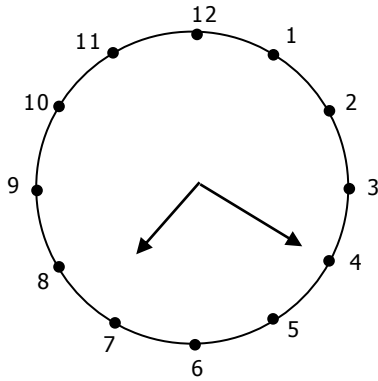
$$\begin{aligned} \checkmark A(\text{sector OPQ}) &= \frac{1}{2} \cdot r^2 \cdot \theta \\ &= \frac{1}{2} \times 144 \times \frac{\pi}{3} \\ &= 24\pi \text{ sq. cm} \end{aligned}$$

$$\begin{aligned} \checkmark \text{ area between the arc PQ and chord PQ} &= A(\text{sector OPQ}) - A(\Delta OPQ) \\ &= 24\pi - 36\sqrt{3} \\ &= 12(2\pi - 3\sqrt{3}) \text{ sq. cm} \end{aligned}$$

SUMS ON CLOCK

Find the degree and radian measure of the angle between the hour hand and the minute hand of a clock at

a) twenty minutes past seven



angle between two consecutive marks

$$= \frac{360}{12} = 30^\circ$$

angle between marks 4 and 7

$$= 3 \times 30^\circ = 90^\circ$$

angle traced by hour hand in 20 minutes

$$= \frac{20}{60} \times 30^\circ = 10^\circ$$

\therefore angle between two hands

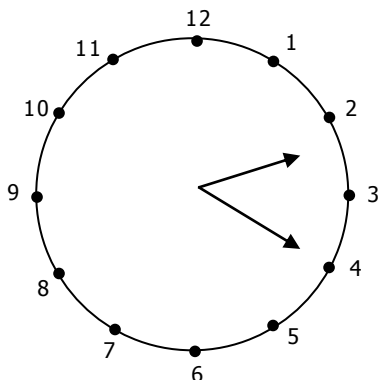
$$= 90 + 10$$

$$= 100^\circ$$

$$= 100 \times \frac{\pi}{180}$$

$$= \frac{5\pi^c}{9}$$

b) twenty minutes past two



angle between two consecutive marks

$$= \frac{360}{12} = 30^\circ$$

angle between marks 3 and 4

$$= 30^\circ = 90^\circ$$

angle traced by hour hand in 20 minutes

$$= \frac{20}{60} \times 30^\circ = 10^\circ$$

\therefore angle between two hands

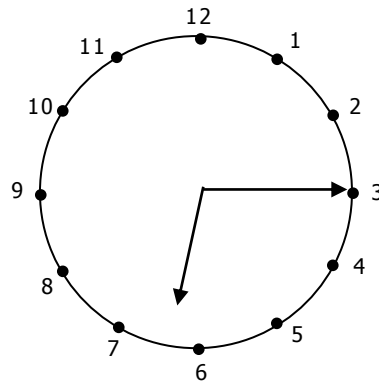
$$= (30 - 10)^\circ + 30^\circ$$

$$= 50^\circ$$

$$= 50 \times \frac{\pi}{180}$$

$$= \frac{5\pi^c}{18}$$

c) quarter past six



angle between two consecutive marks

$$= \frac{360}{12} = 30^\circ$$

angle between marks 3 and 6

$$= 3 \times 30^\circ = 90^\circ$$

angle traced by hour hand in 15 minutes

$$= \frac{15}{60} \times 30^\circ = 7.5^\circ$$

\therefore angle between two hands

$$= 90 + 7.5$$

$$= 97.5^\circ$$

$$= 97.5 \times \frac{\pi}{180}$$

$$= \frac{13\pi^c}{24}$$

COTERMINAL ANGLES :

Angles of different measures having same positions of initial ray and terminal ray are called co – terminal angles

If the two directed angles are co terminal angles then the difference between measures of these two directed angles is **an integral multiple of 360°**

a) 210° , -150°

$$210 - (-150) = 360^\circ$$

the given pair of angles are CO-TERMINAL

b) 330° , -60°

$$330 - (-60) = 390 \text{ is not a multiple of } 360^\circ$$

the given pair of angles are NOT CO-TERMINAL

c) 405° , -675°

$$405 - (-675) = 1080^\circ = 3 \times (360^\circ)$$

the given pair of angles are CO-TERMINAL

d) 1230° , -930°

$$1230 - (-930) = 2160^\circ = 6 \times (360^\circ)$$

the given pair of angles are CO-TERMINAL